

Unit - II

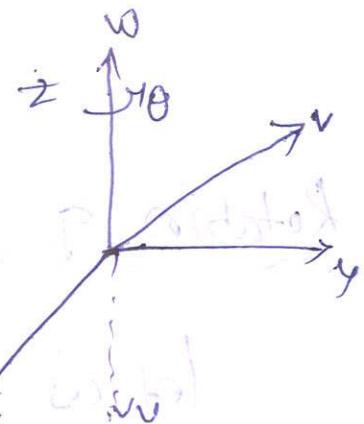
- Motion Analysis:-

Basic Rotation matrix:-

In order to define the orientation of frame L_2 w.r.t. L_1 frame, which is rotating about z -axis is called "Basic Rotation matrix" one can also called by fundamental rotation matrix. Representation of Basic Rotation matrix is one or two $R_z(\theta)$ [Rotating about z -axis]

$$R_z(\theta), R_z(\theta, \theta)$$

Rotating it self by θ of z -axis:-



$$R_z = \begin{bmatrix} \hat{x}, \hat{x} & \hat{x}, \hat{y} & \hat{x}, \hat{w} \\ \hat{y}, \hat{x} & \hat{y}, \hat{y} & \hat{y}, \hat{w} \\ \hat{z}, \hat{x} & \hat{z}, \hat{y} & \hat{z}, \hat{w} \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Principle axis z -axis is rotating it self by θ

of z -axis is above

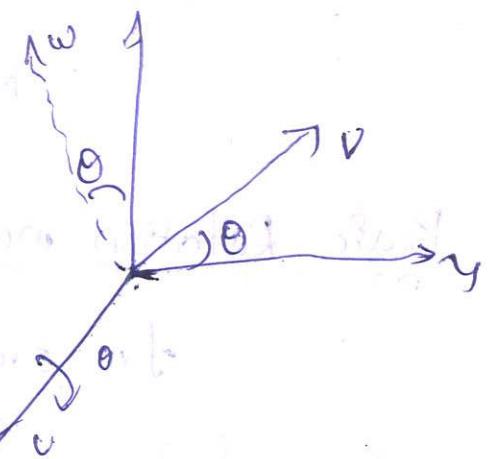
$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

X-axis:-

Similarly

$$R_x(\theta) =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$



$$R_y(\theta)$$

$$R_y(\theta) =$$

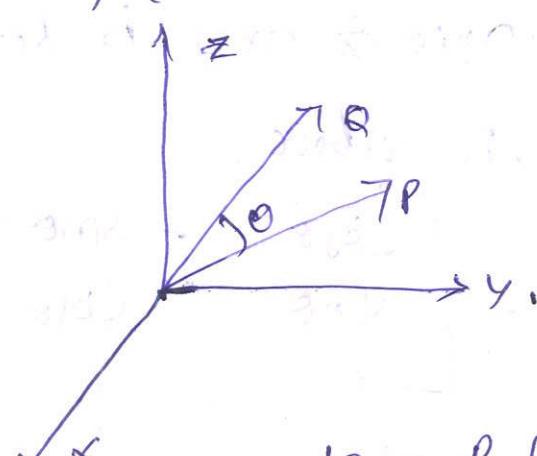
$$\begin{bmatrix} \cos\theta & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) =$$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Rotation of vector:-

Let us consider a vector \vec{P} which is rotating with an angle θ , then the obtained vector is \vec{Q} . Let $R(\theta)$ is the rotation describes the rotation of θ about k -axis i.e. whether it may be x or y or z -axis.



$$\vec{Q} = R_k(\theta) \cdot \vec{P}$$

① Find the coordinates of the vector tube, if the coordinates at point P is $P = [3.0 \ 2.0 \ 1.0]^T$ which is rotating about z-axis. obtain the new position vector on angle 45° rotating.

Soln:-

$$P = [3.0 \ 2.0 \ 1.0]^T$$

$$I_Q = R_z(\theta) \cdot I_P$$

$$I_Q = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_Q = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.0 \\ 2.0 \\ 1.0 \end{bmatrix}$$

$$I_Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.0 \\ 2.0 \\ 1.0 \end{bmatrix}$$

$$I_Q = \begin{bmatrix} \frac{1}{\sqrt{2}} \times 3.0 - \frac{1}{\sqrt{2}} \times 2.0 + 0 \\ \frac{1}{\sqrt{2}} \times 3.0 + \frac{1}{\sqrt{2}} \times 2.0 + 0 \\ 0 + 0 + 1 \end{bmatrix} = \begin{bmatrix} 0.707 \\ 3.53 \\ 1 \end{bmatrix}$$

$$I_Q = \begin{bmatrix} 0.707 \\ 3.53 \\ 1 \end{bmatrix}$$

② Find the coordinate of Q which is of new position vector. The coordinates of P are $[4, 6, 9]^T$ which is rotating about x-axis with an angle θ . So obtain the new position vector.

Soln:-

$$\underline{P} = [4 \ 6 \ 9]^T$$

$$\underline{I}_Q = R_x(\theta) \cdot \underline{I}_P$$

$$\underline{I}_Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \cdot \underline{I}_P$$

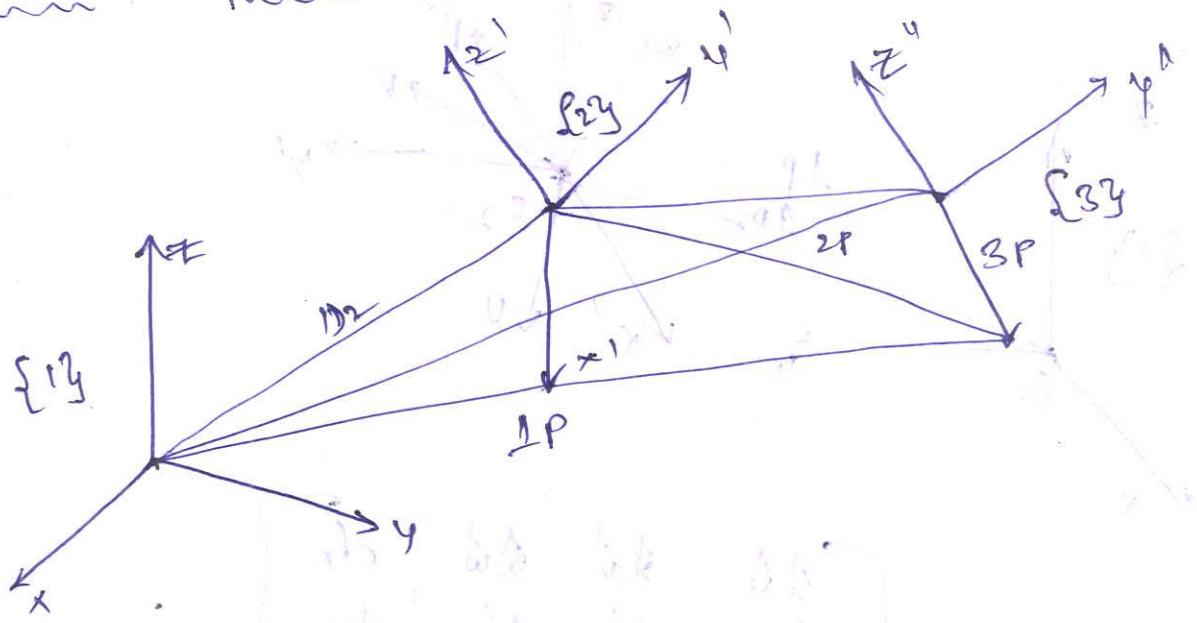
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60^\circ & -\sin 60^\circ \\ 0 & \sin 60^\circ & \cos 60^\circ \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 9 \end{bmatrix}$$

$$\underline{I}_Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 9 \end{bmatrix}$$

$$\underline{I}_Q = \begin{bmatrix} 4 \\ \frac{1}{2} \times 6 - \frac{\sqrt{3}}{2} \times 9 \\ \frac{\sqrt{3}}{2} \times 6 + \frac{1}{2} \times 9 \end{bmatrix} = \begin{bmatrix} 4 \\ -4.794 \\ 9.69 \end{bmatrix}$$

$$\underline{I}_Q = \begin{bmatrix} 4 \\ -4.794 \\ 9.69 \end{bmatrix}$$

Composite rotation matrices



Transformation of vector:-

(i) Rotation of vector

(ii) Translation of vector

(iii) Rotation of vector:-

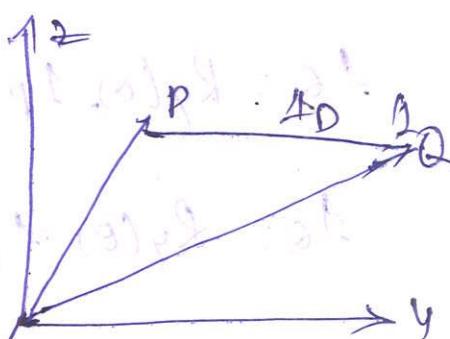
A vector \vec{p} is vector with some angle θ and the obtained vector is $\vec{1q}$.

Mathematically expressing i.e. $\vec{1q} = R_E(\theta) \cdot \vec{1p}$

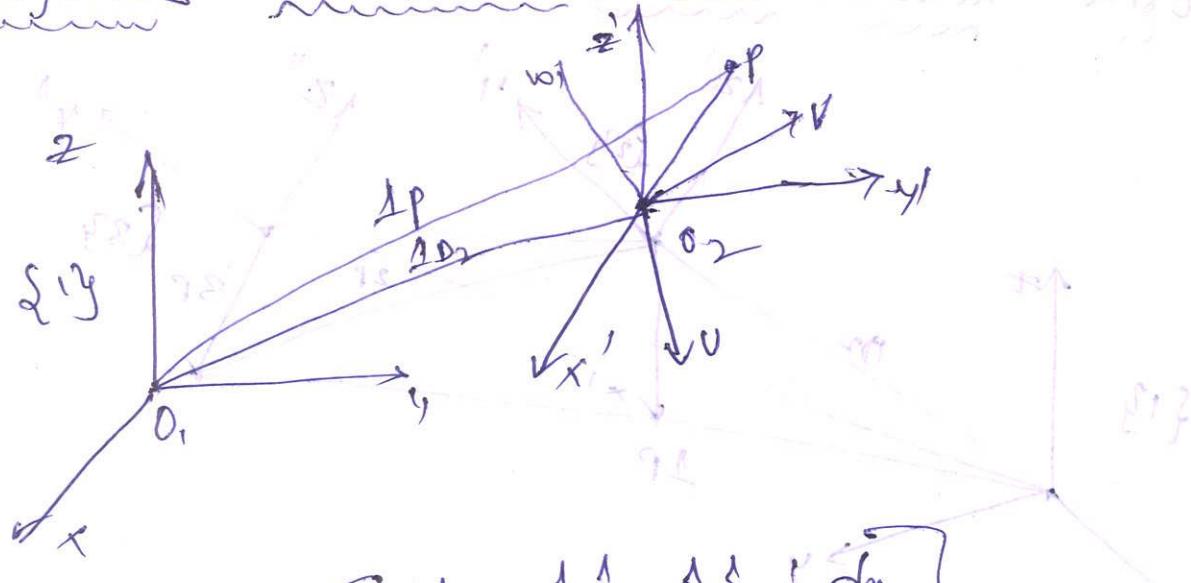
(iv) Translation of vector:-

A vector $\vec{1p}$ which is translated by a distance $1d$ and the obtained new position vector is $\vec{1q}$.

$$\text{M.E } \vec{1q} = \vec{1f}_p + \vec{1d}$$



Homogeneous Transformation Matrix



$$T = \begin{bmatrix} x_1 & x_2 & x_3 & dx \\ y_1 & y_2 & y_3 & dy \\ z_1 & z_2 & z_3 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) rotation about x
(b) rotation about y
(c) rotation about z

$$T = \begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) translation along x
(b) translation along y
(c) translation along z

A homogeneous Transformation matrix is a matrix which describes the position and orientation of frames. It is denoted by T .

- Find Coordinate of Q if the Coordinates of P are $P[4.0 \ 6.0 \ 7.0]^T$ if making an angle $\theta = 45^\circ$ which is rotating about y axis.

Soln

$$1Q = R_y(\theta) \cdot 1P$$

$$1Q = R_y(45^\circ) \cdot 1P$$

$${}^1Q = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 4.0 \\ 6.0 \\ 7.0 \end{bmatrix}$$

$${}^1Q = \begin{bmatrix} \cos 60^\circ & 0 & \sin 60^\circ \\ 0 & 1 & 0 \\ -\sin 60^\circ & 0 & \cos 60^\circ \end{bmatrix} \begin{bmatrix} 4.0 \\ 6.0 \\ 7.0 \end{bmatrix}$$

$${}^1Q = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 4.0 \\ 6.0 \\ 7.0 \end{bmatrix}$$

$${}^1Q = \begin{bmatrix} \frac{\sqrt{3}}{2} \times 4 + \frac{1}{2} \times 7 \\ 6 \\ -\frac{1}{2} \times 4 + \frac{\sqrt{3}}{2} \times 7 \end{bmatrix}$$

$${}^1Q = \begin{bmatrix} \frac{11\sqrt{3}}{2} \\ 6 \\ \frac{5\sqrt{3}}{2} \end{bmatrix}$$

\Rightarrow Find the coordinates of point Q. The position vector which is rotating about z-axis, making an angle 60° if the coordinates of point P in frame 1P are $[6.0 \ 4.0 \ 9.0]^T$

Soln ${}^1Q = R_y(60^\circ) \cdot {}^1P$

$$= \begin{bmatrix} \cos 60^\circ & 0 & -\sin 60^\circ \\ 0 & 1 & 0 \\ \sin 60^\circ & 0 & \cos 60^\circ \end{bmatrix} \begin{bmatrix} 6.0 \\ 4.0 \\ 9.0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 6.0 \\ 4.0 \\ 9.0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} & \frac{\sqrt{3}}{2} & 0 & -\frac{3\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{3}{2} & 0 & \frac{3\sqrt{3}}{2} \\ 0 & 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

about x-axis

$$= \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} \times 6 - \frac{\sqrt{3}}{2} \times 4 \\ \frac{\sqrt{3}}{2} \times 6 + \frac{1}{2} \times 4 \\ 0 \\ 0 \end{bmatrix}$$

and so on (for y and z axis)

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ (rotation about y-axis)}$$

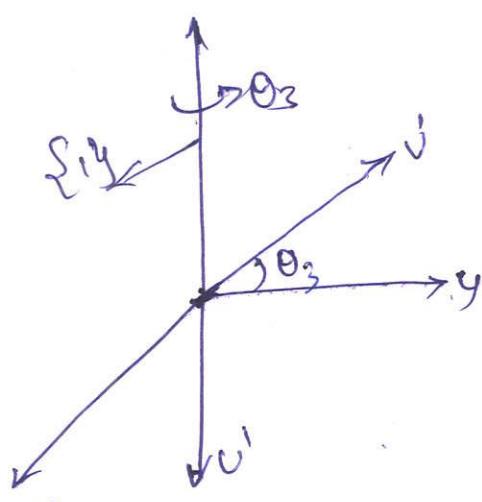
so for p & q taking $\begin{bmatrix} 3\sqrt{2}/2 \\ 3\sqrt{2}/2 \\ 1 \end{bmatrix}$ (rotation about z-axis)

$$= \begin{bmatrix} 0.46 \\ 7.19 \\ 9 \end{bmatrix} \text{ (rotation about z-axis)}$$

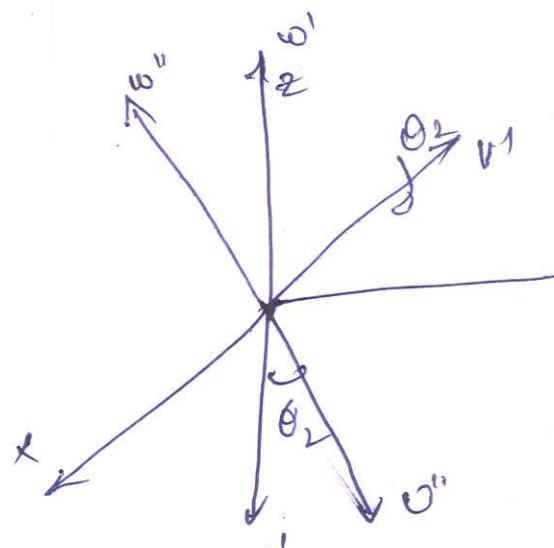
$$\begin{bmatrix} 0.46 \\ 7.19 \\ 9 \end{bmatrix} = \begin{bmatrix} 0.46 \\ -0.46 \\ 7.19 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.46 \\ 7.19 \\ 9 \end{bmatrix} = \begin{bmatrix} 0.46 \\ -0.46 \\ 7.19 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Euler Angle Representation



fig(a)



fig(b)

$${}^1 R_2 = {}^1 R_2(\omega' v' u')$$

$$\Rightarrow {}^1 R_2(\theta_3)$$

$$\Rightarrow {}^1 R_{v'}(\theta_2)$$

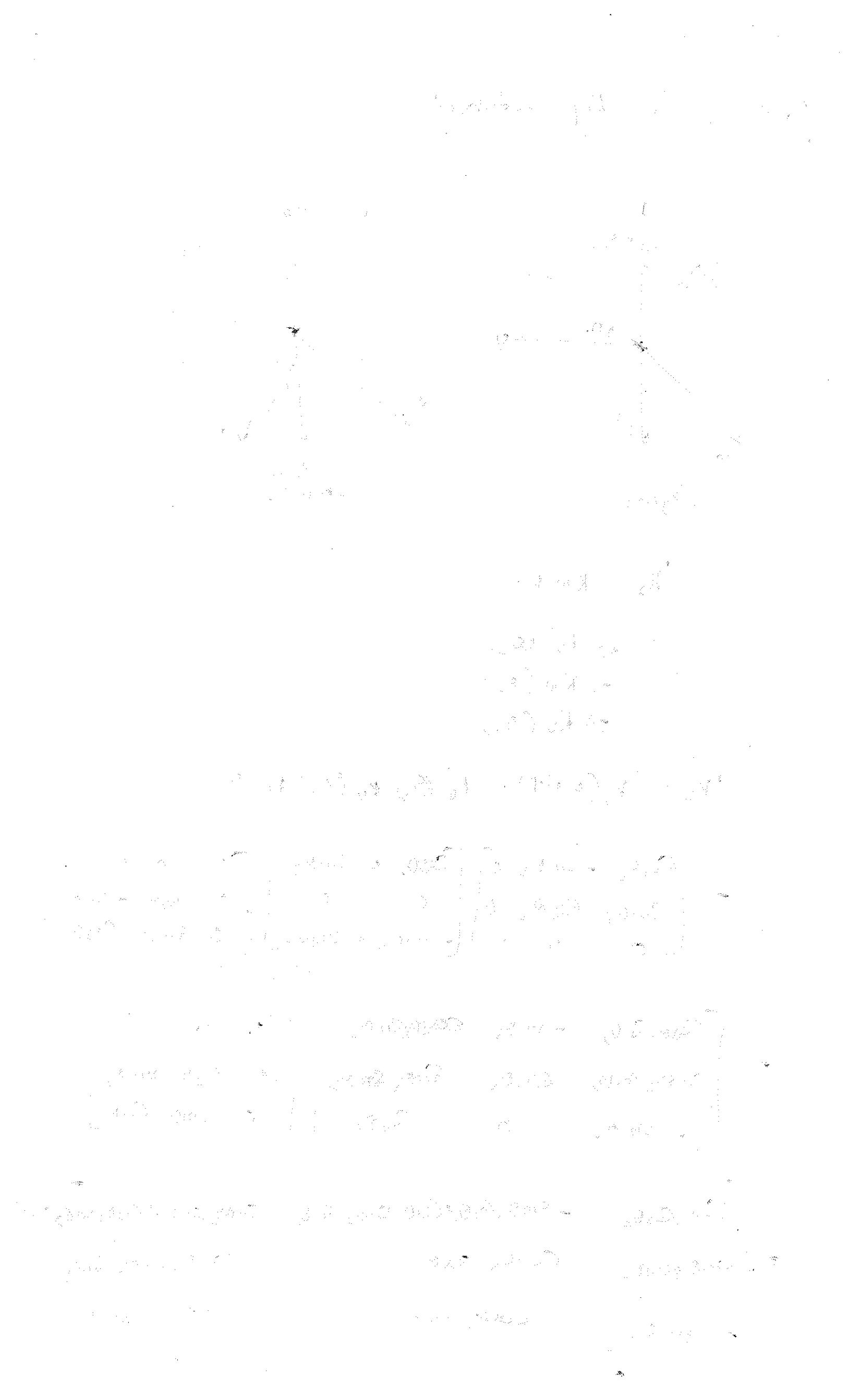
$$\Rightarrow {}^1 R_{u'}(\theta_1)$$

$${}^1 R_2 = {}^1 R_2(\omega' v' u') = {}^1 R_2(\theta_3) {}^1 R_{v'}(\theta_2) {}^1 R_{u'}(\theta_1)$$

$$= \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 \\ \sin \theta_3 & \cos \theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_2 & 0 & \sin \theta_2 \\ 0 & 1 & 0 \\ -\sin \theta_2 & 0 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & -\sin \theta_1 \\ 0 & \sin \theta_1 & \cos \theta_1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta_3 \cos \theta_2 & -\sin \theta_3 & \cos \theta_3 \sin \theta_2 \\ \sin \theta_3 \cos \theta_2 & \cos \theta_3 & \sin \theta_3 \sin \theta_2 \\ -\sin \theta_2 & 0 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & -\sin \theta_1 \\ 0 & \sin \theta_1 & \cos \theta_1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta_3 \cos \theta_2 & -\sin \theta_3 (\cos \theta_1 + \cos \theta_3 \sin \theta_2 \sin \theta_1) & \sin \theta_3 \sin \theta_1 + \cos \theta_3 \sin \theta_2 \cos \theta_1 \\ \sin \theta_3 \cos \theta_2 & \cos \theta_3 \cos \theta_1 & \sin \theta_3 \sin \theta_1 \sin \theta_2 \\ -\sin \theta_2 & \cos \theta_2 \sin \theta_1 & \cos \theta_2 \cos \theta_1 \end{bmatrix}$$

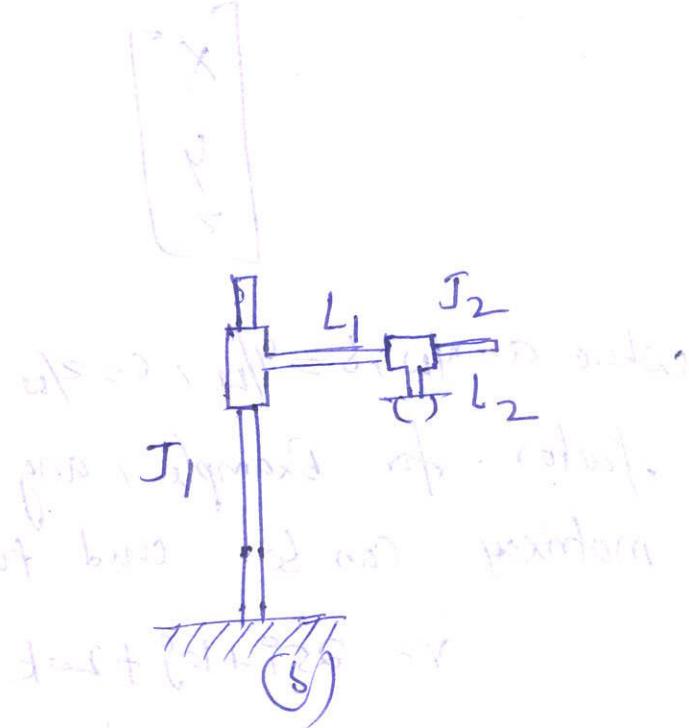
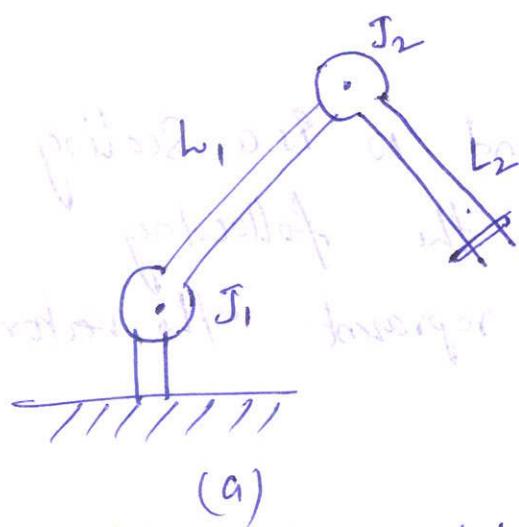


Robot Motion Analysis and Control

Introduction to manipulator kinematics-

In order to develop a scheme for controlling the motion of a manipulator it is necessary to develop techniques for representing the position of the arm at point in time we will define the robot manipulator using the two basic elements . joints and links . Each joint represents 1 degree of freedom . As discussed in Chap. 2 the joints may involve either linear motion or rotational motion between the adjacent links . According to our definitions the links are assumed to be the rigid structures that connect the joints .

Joint are labeled starting from 1 and moving towards the end effector with the base being joint.



Two different two-jointed manipulators : (a) two rotational joints (RR)
 (b) two Linear Joints (LL)

We will also use the symbol L_m to indicate the length of the link in some of our equation derivations early in the chapter. Later in the chapter, we define a term

Homogeneous Transformations and Robot Kinematics

The approach used in the previous section becomes quite cumbersome when a manipulator with many joints must be analyzed. Another, more general method for

solving the kinematic equations of a robot arm makes use of homogeneous transformations. We describe this technique in this section, assuming the reader has at least some familiarity with the mathematics of vectors and matrix. It begins by defining the notation to be used.

A point vector $v = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ can be represented in three dimensional space by the column matrix

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where $a = x/w$, $b = y/w$, $c = z/w$, and w is a scaling factor. For example, any of the following matrices can be used to represent the vector

$$v = 25\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}$$

$$\begin{bmatrix} 25 \\ 10 \\ 20 \\ 1 \end{bmatrix} \text{ or } \begin{bmatrix} 50 \\ 20 \\ 40 \\ 2 \end{bmatrix} \text{ or } \begin{bmatrix} 12.5 \\ 5.0 \\ 10.0 \\ 0.5 \end{bmatrix}$$

The transformation to accomplish a translation of a vector in space by a distance a in the x -direction, b in the y -direction, and c in the z -direction is given by

$$H = \text{Trans}(a, b, c) \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

manipulator path Control :-

In Controlling the manipulator, we are not only interested in the endpoint reached by the robot joints, but also in the path followed by the arm in travelling from one point to another in the workspace.

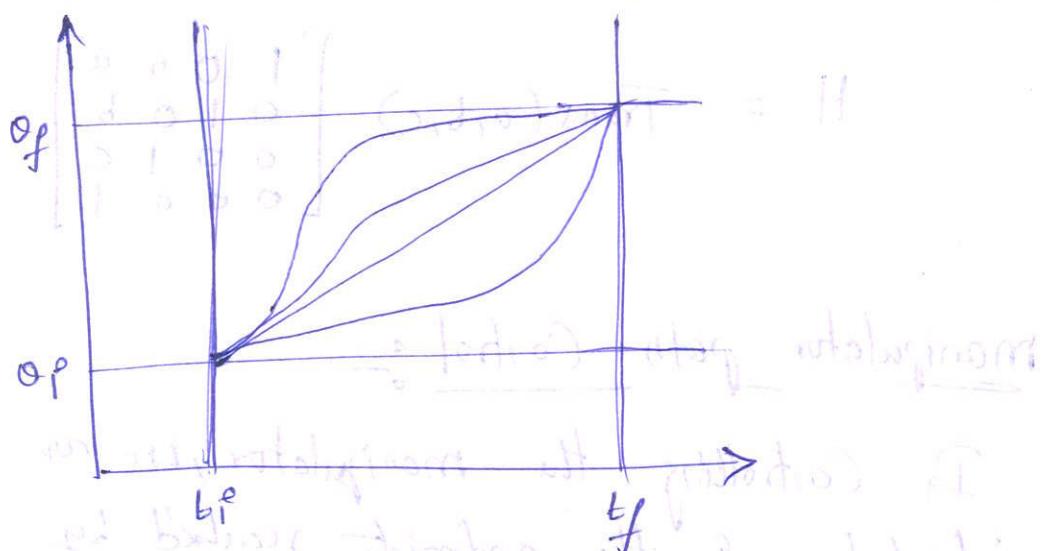
1. motion types:

There are three common types of motion that a robot manipulator can make in travelling from point. They are slow motion, Joint- Interpolated motion, and straight line motion.

Slew motions represent the simplest type of motion. As the robot is commanded to travel from point A to point B, each axis of the manipulator travels as quickly as possible.

Joint - Space Schemes

In this scheme the path or shapes in space and time are written as functions of joint angles. This has the added advantage that there is no continuous correspondence between joint space and Cartesian space. Hence there is no problem of singularity avoidance etc.



Curve connecting initial and final point

A cubic polynomial has the form

$$\theta(t) = a_1 + a_2 t + a_3 t^2 + a_4 t^3$$

The velocity and acceleration can be expressed by

$$\dot{\theta}(t) = a_2 + 2a_3 t + 3a_4 t^2$$

$$\ddot{\theta}(t) = 2a_3 + 6a_4 t$$

When a link moves it starts from rest and stops at rest, hence the initial and final velocity have to be zero at initial time t_i and final time t_f .

$$\dot{\theta}(0) = 0$$

$$\dot{\theta}(t_f) = 0$$

Hence, combining the two velocity constraints and two position constraints (start position and end position) and using eqn (1) we get the following equations:

$$\theta_1 = \alpha_1 t + \beta_1 \quad \text{Eqn 1}$$

$$\theta_f = \alpha_1 t_f + \alpha_2 t^2 f + \alpha_3 t^3 f \quad \text{Eqn 2}$$

$$0 = \alpha_2$$

$$0 = \alpha_2 + 2\alpha_3 t_f + 3\alpha_4 t_f^2$$

Solving the four equations, we have

$$\alpha_1 = \theta_i \quad \theta = \theta_i + \beta_1$$

$$\alpha_2 = 0 \quad \theta = \theta_i + \beta_1$$

$$\alpha_3 = \frac{3}{t_f^2} (\theta_f - \theta_i)$$

$$\alpha_4 = -\frac{2}{t_f^3} (\theta_f - \theta_i)$$

Robot Dynamics:-

Accurate control of the manipulator requires precise control of each joint. The control of the joints depends on knowledge of the forces that will be acting on the joints and the inertias reflected at the joints (the masses of the joints and links of the manipulator) while these forces and masses are relatively easy to determine for a single joint, it becomes more difficult to determine them as the complexity of the manipulator increases.

We will explore three issues concerning the two-axis manipulator developed earlier. Our purpose is to introduce this problem area to the reader rather than to analyze its complexities in detail.

Static Analysis:

Let us begin by considering the forces required by the joints to produce a force on each link.

We get

and

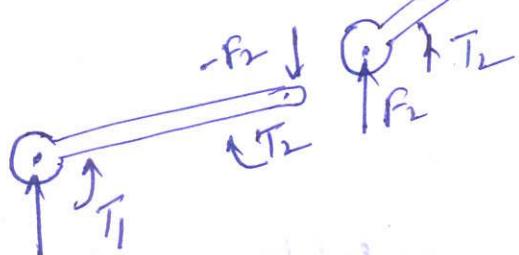
$$F_1 - F_2 = 0 \quad \beta = p^0$$

$$F_2 - F = 0 \quad \alpha = p^0$$

that is

$$F_1 = F_2 = F \quad \beta = p^0$$

$$(p^0 - p^0) \frac{L}{L} = p^0$$



$$(p^0 - p^0) \frac{L}{L} = p^0$$

Now F_1 requires clockwise torque about joint 1.

Joint 1 is left of joint 2. Joint 2 is right of joint 1.

Force at the left end of the link is clockwise.

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Joint 1 is left of joint 2. Joint 2 is right of joint 1.

Two link arm force and torque

The torque are the vector cross-products of the forces and the link vectors or developed in eqs (4.1) and (4.2) so that

$$\vec{T}_1 = \vec{r}_1 \times \vec{F} \quad (4.54)$$

$$\vec{T}_2 = \vec{r}_2 \times \vec{F} \quad (4.55)$$

therefore we can state

$$\vec{T}_1 = (\vec{r}_1 + \vec{r}_2) \times \vec{F}$$

If $\vec{F} = (F_x, F_y)$ then eq (4.57) can be written as

$$\vec{T}_1 = [L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2), [L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)]] \times (F_x, F_y)$$

since the vector cross-product $(a, b) \times (c, d)$ is $ad - bc$ we get

$$T_1 = [L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)] \times F_x - [L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)] \times F_y$$

and

$$T_2 = L_2 \cos(\theta_1 + \theta_2) \times F_x - L_2 \sin(\theta_1 + \theta_2) \times F_y$$

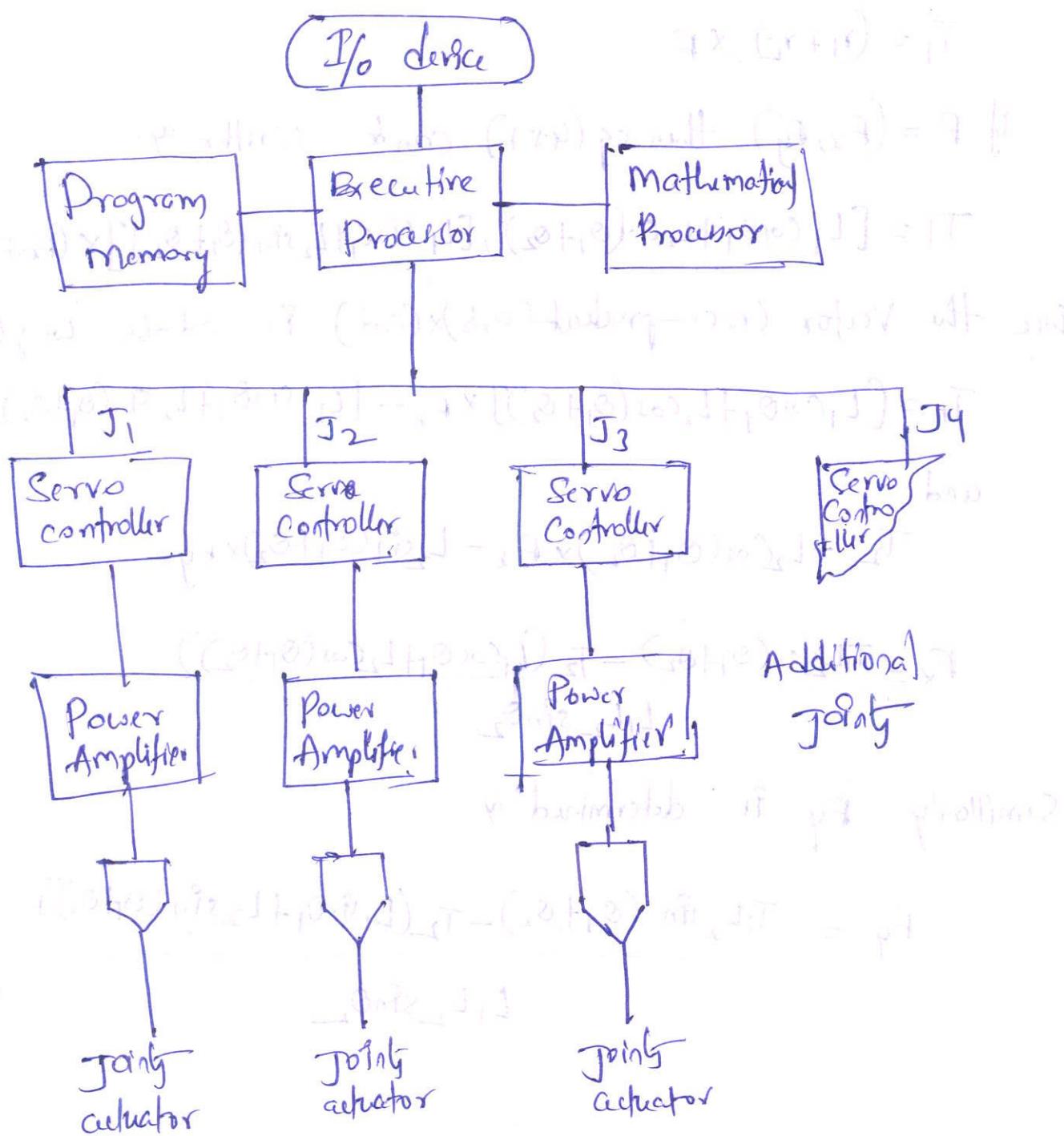
$$F_x = \frac{T_1 L_2 \cos(\theta_1 + \theta_2) - T_2 (L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2))}{L_1 L_2 \sin\theta_2}$$

Similarly F_y is determined by

$$F_y = \frac{T_1 L_2 \sin(\theta_1 + \theta_2) - T_2 (L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2))}{L_1 L_2 \sin\theta_2}$$

Configuration of a robot Controller

If the control requirements outlined in Chap. 3 here and the present chapter are combined, we can develop a general configuration for a robot controller. The elements needed in the controller include just servo controllers, joint power amplifiers, mathematical processor.



General robot Controller element